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From the theory of the form $x^2 + y^2$, the general solution of (1) is given by

$$r + z = m(a^2 + b^2), \qquad r - z = m(c^2 + d^2),$$
$$x = m(ac + bd), \qquad y = m(ad - bc),$$

where at most one of the integers a, b, c, d is zero, and either m or $a^2 + b^2 + c^2 + d^2$ is even. It follows that

$$r = \tfrac{1}{2}m(a^2 + b^2 + c^2 + d^2), \qquad z = \tfrac{1}{2}m(a^2 + b^2 - c^2 - d^2).$$

By permutation of a, b, c, d we obtain six solutions (generally distinct) with r in common from which three may be selected with no other value in common.

Thus, if $m = 2$, $a = 5$, $b = 3$, $c = 1$, $d = 2$, we get for x, y, z the sets of values

$$22, 14, 29;\ 34,\ 2, 19;\ 26, 26, 13;$$
$$26,\ 2, 29;\ 22, 26, 19;\ 34, 14, 13;$$

of which the first or last three are entirely distinct, and in each case the sum of the squares is 39^2.

Evidently, by taking for r a value having various divisors expressed in the form $a^2 + b^2 + c^2 + d^2$, we can obtain any desired number of rectangular parallelepipeds with integral sides and a common integral diagonal.

Also solved by ELIJAH SWIFT, B. F. YANNEY, H. C. FEEMSTER and J. L. RILEY.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence.

DISCUSSIONS.

I. RELATING TO THE DERIVATION OF THE DISTANCE FORMULA.

By H. T. BURGESS, University of Wisconsin.

The derivation of the distance formula independent of Hesse's "normal form" as presented by Mr. Mathews on page 476 of the December MONTHLY appeals to me as an excellent idea. Hesse's form in this connection may well be relegated to Professor Miller's collection of "Obsoletes."[1]

Since the form $y = mx + b$ includes all lines not parallel to the Y-axis and is the only one ever needed in ordinary straight-line problems, I suggest the derivation below as a possible alternative.

Given a point $P = (x_1, y_1)$ and a line $y = mx + b$, to find the distance d from the point to the line.

Write the equation of the parallel through P in the form $y = mx + b'$. When a drawing is made, it is obvious that, numerically,

$$d = (b' - b)\cos\alpha = \frac{b' - b}{\sqrt{1 + m^2}},$$

where $\alpha = \arctan m$.

If one must be conventional, we have

$$d = \frac{b' - (\pm b)}{\pm\sqrt{1 + m^2}} = \frac{y_1 - mx_1 - (\pm b)}{\pm\sqrt{1 + m^2}}$$

to take care of the sign of d.

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[1] *Cf.* pp. 453–456 in the December, 1917, MONTHLY.